智慧型感測器及網路架構在橋樑系統
健康診斷及損壞評估之研發

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摘要

本研究在建置一套以無線感測器為主之量測及分析系統，以收集橋梁之振動資料，以即時性分析軟體，對橋梁結構進行健康診斷，並即時予以顯示預警值，及橋梁結構現階段之動態特性。亦再次採用 WiMMS (Wireless Monitoring Module system)進行以無線及多振動點之方式對橋梁之微振動進行量測。 配合對橋梁之微振量測結果，用以建立更可靠之橋梁微振分析方法。本研究並同時進行重車引致橋梁振下，對所施加於橋梁下之外力進行識別，以了解車輛行進中對橋梁所施加之外力。

Abstract

This research is to develop a wireless sensing system to collect the ambient vibration of bridge structure (using velocity sensors) and the traffic-induced vibration of bridge structure (accelrometers). Based on the collect vibration signals from different types of measurement, two different research topics are developed. From the ambient vibration signal of bridge the mode shapes and modal frequency of the bridge structure (Gi-Lu bridge) can be identified. From the traffic-induced vibration of bridge the input force (traffic) is identified.

ResearchTopic I:

Input Force Identification From the Measurement of Traffic-Induced Vibration of Bridge

1. INTRODUCTION

Accurate characterization of input excitation forces acting on a structure during operation is significant for designing, controlling and diagnosing a system. In some circumstances, input excitation forces can be directly measured using load or acceleration transducers. However, for some physical and mechanical systems, direct measurements of input excitation forces are difficult to be realized due to very large magnitudes of forces or installation obstacle of load transducers. Therefore, an alternate method to reconstruct input excitation forces is needed.

Conventional inverse method for input excitation forces are well known as a de-convolution problem from either time domain or frequency domain. The objective of the conventional inverse method is to deconvolve the matrix equation to produce an estimate of an input forces based on the structural response and the impulse response. For fields of seismic and signal processing, great deals of research have been applied to solve the convolution matrix equation in time domain using methods such as least-squares**.** However, it is well known that the conventional inverse method generally tends to represent an ill-posed problem because of small changes in measurement data can contribute to large changes in the estimate of input excitation forces. To treat the ill-condition problems, some regularization techniques have been demonstrated. Busby and Trujillo (1997) used the least-squares minimization with regularization, which involved adding a term to smooth and stabilize the solutions; while Kammer (1998) proposed a regularization technique with Markov parameters to produce an estimate of the corresponding input excitation forces.

In addition to conventional inverse method, the Kalman filter based tracking approach has been studied and developed for the identification of input excitation forces. Singer (1970) augmented the Kalman filter with the target acceleration equation represented by a first-order autoregressive process. Chan et al. (1979) proposed a Kalman filter based tracking scheme with input estimation and used a detector to guard against automatic updating of the simple Kalman filter. Ma (1998) proposed an application of the Kalman filter to determine the impulse loads of a lumped-mass system in a numerical scheme. Liu et al. (2000) used the Kalman filter and least-squares estimator to determine the input excitations of a cantilever plate. The efficiency and robustness of a regularization scheme of the proposed algorithm using the Kalman filter and recursive least-squares were proven better than the conventional methods (Ma et al. 2003).

In this study, a method using the Kalman filter and recursive least-squares estimator proposed by Liu et al. (2000) is employed. The relationship between the residual innovation and the input excitation forces are estimated using the Kalman filter based on the measured structural responses while least-squares method with a recursive estimator is employed to update the estimation in the sense of real-time computation. Application of proposed approach will be conducted on field test data collected from BiLin Bridge to study the characteristics of traffic loadings induced by commercial vehicles.

2 THEORETICAL BACKGROUND

2. 1 State Equations of the system

For a n degree-of-freedom lumped-mass system, the equation of motion can be written as follows:

$$M\ddot{Y}(t) + C\dot{Y}(t) + KY(t) = F(t) \tag{1}$$

where M denotes the $n \times n$ mass matrix, C is the $n \times n$ damping matrix, K is the $n \times n$ stiffness matrix, $F(t)$ is the $n \times 1$ input force vector, $\ddot{Y}(T)$, $\dot{Y}(t)$ and $Y(t)$ are $n \times 1$ vectors of acceleration, velocity and displacement, respectively.

To estimate the states of linear system through the Kalman filter, the transformation of the equations of motion is performed. The dynamic system with n degrees of freedom will be represented by $2n \times 1$ state vectors $X(t) = [Y(t) \quad \dot{Y}(t)]$. According to equation (1), the continuous-time state and measurement equations can be written as follows:

$$\dot{X}(t) = A_c X(t) + B_c F(t) \tag{2}$$

$$Z(t) = HX(t) \tag{3}$$

Equation (2) and (3) are discretized over time interval of length Δt, and associated with process noise involving statistical description of the system noise and uncertainty in the dynamic models. The equation (2) becomes as follows:

$$X(k+1) = AX(k) + BF(k) + \Gamma w(k) \tag{4}$$

where $X(k)$ represents the state vector, A is the state transition matrix, B is the input matrix, Δt is the sampling interval, and $w(k)$ is the noise vector which is assumed to be zero mean and white with variance $E[w(k)w^T(j)] = Q\delta_k$, where Q is the process noise covariance matrix and δ_k is the Kronecker delta.

Consider the statistical description of the measurement noise, equation (3) becomes:

$$Z(k) = HX(k) + v(k) \tag{5}$$

where $Z(k)$ denotes the observation vector and $v(k)$ represents the measurement noise vector. In addition, $v(k)$ is assumed to be zero mean and white noise. The variance of $v(k)$ is given by $E[v(k)v^T(j)] = R\delta_k$, where R is the measurement noise covariance matrix.

2.2 The Recursive Input Estimation Approach

The unknown input excitation forces derived in the previous section can be estimated from measurement responses of systems through an inverse method, which is composed of two parts: the Kalman filter and a recursive least-squares algorithm.

The Kalman filter, which is the implementation of a predictor-corrector type estimator, is optimal in the sense that it minimizes the estimated error covariance when some presumed conditions are met. The equations of the Kalman filter are shown in Table 1:

The implementation of second part of input excitation forces is recursive least-squares algorithm, which provided a recursive relationship between a residual innovation resulting from the Kalman filter and input excitation forces. The equations of the recursive least-squares algorithm are shown in Table 2:

Consequently, the input excitation forces can be estimated through the following steps:

1 Using system identification techniques to derive the system model and measure the dynamic responses of system.

2 Using the Kalman filter equations to obtain the innovation covariance $S(k)$, innovation $\bar{Z}(k)$ and Kalman gain $K_a(k)$.

3. Using the recursive least-squares algorithm to estimate the input forces.

3. APPLICATIONS TO VEHICLE-BRIDGE INTERACTION STUDY

A Field test was conducted on one of spans of BiLin Bridge, which is located in Shihlin district of Taipei city, Taiwan. Three accelerometers were installed on the bridge, dividing four equal space of the 28m span. The installation of three accelerometers can be treated in the sense as a model of 3 degree-of- freedom system suffered from traffic loading induced by vehicles in each degree of freedom. Following the steps of input estimation process, the bridge properties were predetermined using stochastic subspace technique. The ambient responses collected from three accelerometers were used for identification of system properties. The identified three modes are 4.27 Hz, 14.45 Hz and 15.27 Hz.

Based on the predetermined system properties and measurements, present approach of input force estimation was implemented. Three different measurement levels were conducted to study the characteristics of traffic loadings induced by various types of commercial vehicles. Consequently, identification results of input forces are shown in Figure 1.

When a vehicle passes over a bridge, certain impact amplification effect will be induced on the bridge, then, induced dynamic deflections of the bridge will affect the vertical motions of vehicles. Therefore, the interaction between the bridge and moving vehicle is concerned as an issue of iterative nature. Due to the vehicle-bridge interaction, fourier spectrum of identified vehicle forces is governed by three extent sets of driving frequencies, vehicle frequencies and bridge frequencies. From Figure 2, two sets of frequencies were observed that the bridge frequency has rather highly visibility than the one of vehicles, while the low-components between 0.1Hz-0.4Hz of driving frequencies was obscured to be observed. Moreover, observation on the identified individual force from Figure 3 reveals that vehicle frequencies have less visibility compared with the one of bridge as measurement level increases. The results might be attributed from the forces of suspension system of vehicles. Further verifications on the effects of vehicle damping induced from suspension system have been confirmed by Yang and Lin that the existence of vehicle damping will caused a significant drop in the vehicle frequency component, but influence on the bridge frequency is generally small. Therefore, it provides a point of view that the more heavy traffic loading induced by vehicles, more effects of vehicle damping will contribute, consequently, less visibility of the vehicle frequency from higher scale measurements.

4. CONCLUSIONS

An identification method for estimating the time varying excitation force acting on a structural system based on its measurement response is presented in this study. The method employs the simple Kalman filter to establish a regression model between the residual innovation and the input excitation forces. Based on the regression model, a recursive least-squares estimator is proposed to identify the input excitation forces incorporating measurement noise and modelling error. The practicability and accuracy of present approach have been pre-evaluated by numerical and experimental verifications. The results conclude that the present approach has robustness to deal with the environmental disturbances using the simple Kalman filter and recursive least square with the additional fading factor. Different measurement level of bridge response was conducted. Both characteristics of time domain and frequency domain of identified forces were studied. Interactions between bridge and vehicles were observed from the identified vehicle forces as well as the effects of vehicle damping induced from suspension system of vehicle.

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Table 1: Traditional Kalman filter estimation procedures.

1. State prediction	$\bar{X}(k/k-1) = A\bar{X}(k-1/k-1)$
2. State prediction covariance	$P(k/k-1) = AP(k-1/k-1)A^T + BQB^T$
3. Calculate Innovation Covariance	$S(k) = HP(k/k-1)H^T + R$
4. Estimate Kalman Gain	$K_a(k) = P(k/k-1)H^T S^{-1}(k)$
5. Updated state covariance	$P(k/k) = [I - K_a(k)H]P(k/k-1)$
6. Calculate Innovation	$\bar{Z}(k) = Z(k) - H\bar{X}(k/k-1)$
7. Updated state estimation	$\bar{X}(k/k) = \bar{X}(k/k-1) + K_a(k)\bar{Z}(k)$

Table 2: Recursive least-squares algorithms for input force identification.

1. Sensitivity matrices:	$B_s(k) = H[AM_s(k-1) + I]B$ and $M_s(k) = [I - K_a(k)H][AM_s(k-1) + I]$
2. Correction gain for the updating input forces	$K_b(k) = \gamma^{-1}P_b(k-1)B_s^T(k)[B_s(k)\gamma^{-1}P_b(k-1)B_s^T(k) + S(k)]^{-1}$
3. Error covariance of the estimated input forces	$P_b(k) = [I - K_b(k)B_s(k)]\gamma^{-1}P_b(k-1)$
4. Estimated input forces	$\hat{F}(k) = \hat{F}(k-1) + K_b(k)[\bar{Z}(k) - B_s(k)\hat{F}(k-1)]$

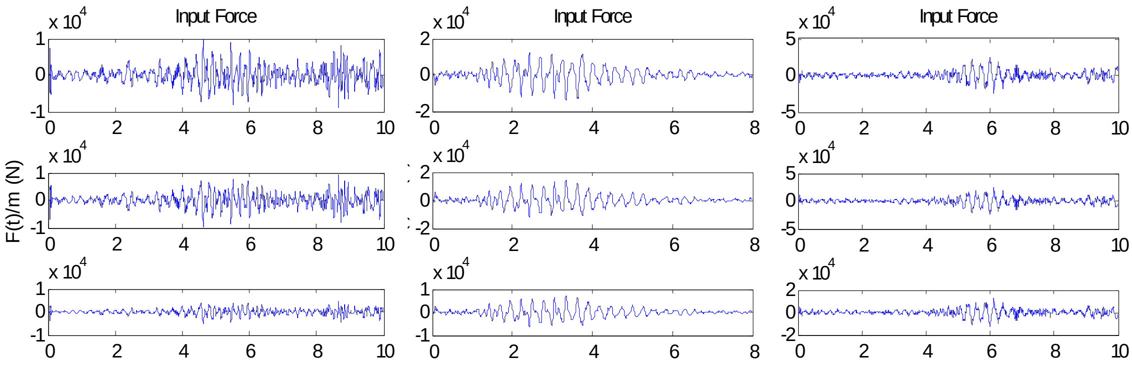


Figure 1: Identified input forces (driving forces)

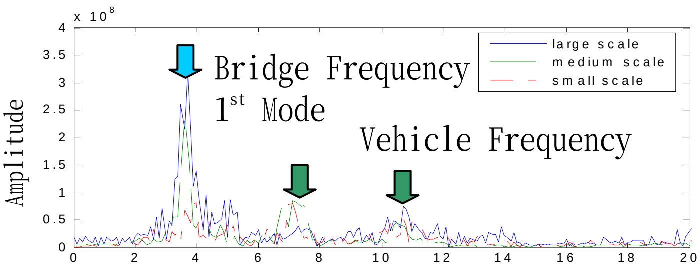
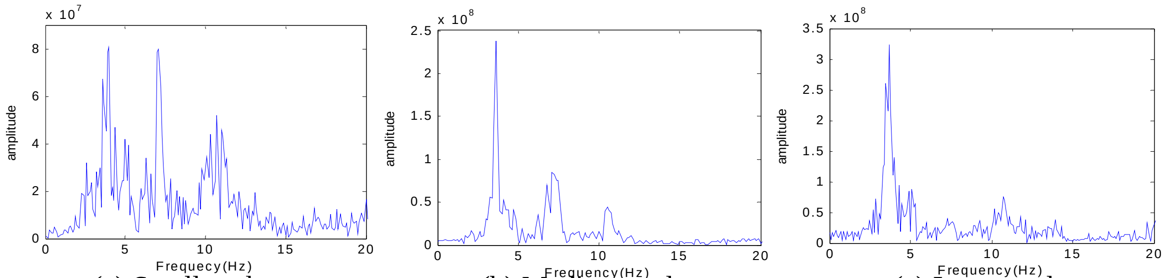


Figure 2: Fourier spectrum of identified vehicle driving forces.



(a) Small scale (b) Medium scale (c) Large scale

Figure 4: Individual Fourier spectrum of identified vehicle driving forces.

Research Topic II

Output-Only Modal Identification of a Cable-Stayed Bridge Using Wireless Monitoring Systems

1. INTRODUCTION

Accurate analysis of both the aerodynamic stability and the earthquake response of cable-stayed bridges often require knowledge of the structure’s dynamic characteristics, including modal frequencies, mode shapes and modal damping ratios. Conducting full-scale dynamic testing is regarded as one of the most reliable experimental methods [1]. Such tests serve to complement and enhance the development of analytical techniques and models that are integral to analysis of the structure over its operational life, however, there is comparatively less information available on full-scale dynamic testing of cable-stayed bridges. A simpler method for determination of the dynamic characteristics of structures is through the use of ambient vibration measurements. In output-only characterization, the ambient response of a structure is recorded during ambient influence by means of highly-sensitive velocity or acceleration sensing transducers. The concurrent development of novel sensing technologies, and high-speed computing and communication technologies currently allows the engineering community to measure and evaluate ambient structural vibrations quickly and accurately.

The use of wireless communications in lieu of wires within a structural monitoring system was initially proposed by Straser and Kiremidjian [2] as a means of reducing installation costs in large-scale civil structures. Recently, Lynch *et al.* has extended their work to include computational microcontrollers in the hardware design of wireless sensors so that various system identification and damage detection algorithms can be embedded for local execution by the sensor [3-4]. To date, a handful of bridges and buildings have been instrumented with wireless monitoring systems including the Alamosa Canyon Bridge (New Mexico), Geumdang Bridge (Korea), WuYuan Bridge (China), Voigt Bridge (California) and a historic theater in Detroit, Michigan [3]. These extensive field studies attest to the accuracy and reliability of wireless sensors in traditional structural monitoring applications.

The purpose of this study is to employ a rapid-to-deploy wireless structural monitoring system prototyped by Wang, *et al.* [6] for monitoring long-span bridges during ambient excitation conditions.Therefore, this study will focus on the experimental determination of the dynamic properties of the newly retrofitted Gi-Lu cable-stayed bridge (Nantou County, Taiwan) using ambient vibration responses recorded by a wireless structural monitoring system. The wireless monitoring system consists of a distributed network of wireless sensors in direct communication with a high-performance data repository where data is stored and analyzed. To extract the bridge modal characteristics, both the frequency domain decomposition (FDD) and stochastic subspace identification (SSI) methods were embedded in the central repository to autonomously identify the dynamic properties of the bridge. The paper concludes with a discussion on the results obtained using the wireless monitoring system, including observation of the interaction between cable and deck vibrations.

2. AMBIENT VIBRATION MEASUREMENTS

The cable-stayed bridge selected for this study is the Gi-Lu Bridge, which is a modern pre-stressed concrete cable-stayed bridge that crosses the Juosheui River. The bridge has a single pylon and two rows of harped cables on each side. The bridge deck consists of a box girder section 2.75 m deep and 24 m wide and is rigidly connected to the pylon; the deck spans 120 m on each side of the pylon. On September 21, 1999, a significant earthquake (Chi-Chi Earthquake) with $M_L = 7.3$ struck the central part of Taiwan. Gi-Lu Bridge was subjected to very strong ground motions resulting in the damage of several of the bridge's critical structural elements. Reconstruction work undertaken to repair the bridge damage was completed at the end of 2004. At that time, the bridge owner elected to develop an experimentally-calibrated finite element model of the bridge so that bridge safety could be verified over the bridge operational lifespan. To accurately calibrate the model, an ambient vibration survey was conducted to extract the modal characteristics of the bridge.

Instrumentation and data acquisition: The instrumentation installed in the bridge consisted of the following components: (1) *Wireless sensors*: twelve wireless sensors each containing a four-channel sensor interface with high-resolution analog-to-digital conversion are used; (2) *Transducers*: interfaced to each wireless sensor node is a highly sensitive Tokyo Sokushin VSE-15 velocity meter whose sensitivity constant is 0.25Volt/kine (where 1 kine is equal to 1 cm/s); (3) *Data repository computer*: one high-performance laptop computer with a wireless modem serves as the core of the system responsible for triggering the system, archiving recorded response data, and autonomously extracting the bridge modal characteristics.

Due to the limited number of sensing nodes available (only 12 wireless sensor-velocity meter pairs), the wireless monitoring system is reconfigured during testing to achieve three different test configurations: (1) *Test 1*: Ten wireless sensor-velocity meter pairs are installed along the bridge deck to record its vertical vibration at locations denoted as V01 through V10 as shown in ***Fig. 1a***; (2) *Test 2*: The ten wireless sensor-velocity meter pairs used during Test 1 are reoriented to record the deck's transverse vibration (denoted as H01 through H10 in ***Fig. 1a***); (3) *Test 3*: All twelve wireless sensors are installed on one side of the bridge to simultaneously record the cables and deck vibrations at sensor location T01 through T12 (***Fig. 1b***). Data was sampled at 100 points per second on each channel to provide good waveform definition. The analog voltage output of the velocity meter was converted to a digital signal with 16-bit resolution by each wireless sensor. The synchronized time-histories collected by the wireless monitoring system were wirelessly broadcasted to the high-performance laptop computer serving as the monitoring system's sole data repository.

Wireless sensors for structural monitoring: A network of wireless sensing units, developed by Wang *et al.* [6] were installed upon the Gi-Lu Bridge in lieu of a traditional tethered structural monitoring system which are known to suffer from high-costs and laborious installations. The design of the wireless sensing unit includes three major subsystems: the sensing interface, the computational core, and the wireless communication system. The sensing interface is responsible for converting analog sensor outputs spanning from 0 to 5V on four independent channels into 16-bit digital formats. Any sensing transducer can be interfaced to the wireless sensing unit with accelerometers, strain gages, displacement transducers and velocity meters all previously interfaced. The digital data is then transferred to the computational core by a high-speed serial peripheral interface (SPI) port. Abundant external memory (128 kB) is associated with the computational core for local data storage (up to 64,000 sensor data points can be stored at one time) and analysis. For reliable communication on the wireless channel, the Maxstream XStream wireless modem operating on the 2.4 GHz wireless band is selected. The outdoor communication range of the modem is up to 300 m line-of-sight which is sufficient for most large-scale civil structures. To enhance the range and reliability of communication in this study, directional antennas (D-link) were attached to each sensing unit to concentrate the energy associated with the wireless transmission in a concentrated beam pointed towards the central data repository.

2. STOCHASTIC SUBSPACE IDENTIFICATION VERSUS FREQUENCY DOMAIN DECOMPOSITION

In this study, the stochastic subspace identification (SSI) method to identify modeal cahractersistics, as originally presented by Van Overschee and De Moor [7], is adopted to identify a stochastic state space model of the Gi-Lu bridge using output-only measurements recorded by the wireless monitoring system. An extension of the original SSI method that does not require output covariance was proposed by Peeters and de Roeck [8] as the reference-based SSI method.

***Stochastic Subspace Identification*:** A discrete-time stochastic state-space model is written as:

$$\begin{aligned} x_{k+1}^s &= A x_k^s + w_k \\ y_k^s &= C x_k^s + v_k \end{aligned} \tag{1}$$

where the superscript "s" denoting "stochastic" since the system is assumed to be excited by a stochastic component. The SSI method is used to identify the system matrices, A and C, from the system output measurements, y_k^s (*i.e.* ambient vibration measurements). The procedures for SSI are briefly introduced using ***Fig.2*** of the flow chart of SSI method.

Frequency Domain Decomposition (FDD) method: A second modal estimation method is adopted in this study termed the frequency domain decomposition (FDD) method [9]. In this identification method, the first step is to estimate the power spectral density (PSD) matrix from the measurements and then decomposed at $\omega = \omega_i$ by taking the SVD of the matrix:

$$\hat{G}_{yy}(j\omega_i) = U_i S_i U_i^T \tag{2}$$

where the matrix $U_i = [u_{i1}, u_{i2}, \cdots, u_{im}]$ is a matrix holding the singular vectors u_{ij}, and S_i is a diagonal matrix holding the scalar singular values s_{ij}. If only the k^{th} mode is present at the selected frequency, ω_i, then there will be only one singular value in Eq. (2). Thus, the first singular vector u_{i1} would then serve as an estimate of the k^{th} mode shape, $\hat{\phi} = u_{i1}$. To implement the FDD method, some prior knowledge of the modal frequencies is required; traditional peak-picking methods can be adopted using the frequency response function of the system calculated for each system output. An advantage of the FDD method is that if two modes are closely spaced and can be identified previously (*e.g.* using the aforementioned SSI method), they can be identified based upon multiple singular values present at a selected frequency.

3. ANALYSIS OF BRIDGE AMBIENT VIBRATION DATA: DYNAMIC PROPERTIES OF THE DECK AND CABLES

3.1. Data analysis using all output measurements from the deck simultaneously:

Using time history data collected during Test 1 and 3, the modal frequencies of the bridge determined by the high-performance data repository executing the reference-based SSI method are tabulated in ***Table 1***. In addition, the first ten bridge deck mode shapes determined by the SSI method during Test 1 are shown in ***Fig. 5a***. Using the FDD method, a second set of identified mode shapes of the bridge are determined and plotted on the same figure (***Fig. 5a***) for comparison. The estimated mode shapes of the bridge deck using both methods are consistent. Using the time history data collected during Test 2, the identified mode shapes of the bridge deck in the transverse direction are also shown in ***Fig. 5b***. Again, excellent agreement between SSI- and FDD-derived mode shapes is evident.

3.2. Data Analysis from the interaction between deck and cable vibration:

After data has been collected, Fourier analysis is applied on the same data set off-line. ***Fig. 6a*** and ***Fig. 6b*** plot the Fourier amplitude spectra of both the horizontal and vertical ambient vibration of the two instrumented bridge cables (R13 is a short cable and R27 is a long cable). From the Fourier amplitude spectrum of cable vibration data, there are several dominant frequencies in the lower frequency range which belong to the deck vibration modes and not the cable itself. This can be better observed from ***Fig. 6c*** where the Fourier amplitude spectrum of the cable vibration data and the deck vibration data are plotted on the same graph. By comparing the identified dominant frequencies using the SSI and off-line Fourier methods, one can clearly observe that the close interaction between the deck and cable vibrations, particularly in the lower frequency range (0 – 2 Hz).

4. CONCLUSIONS

1. Less effort and man-power were required during the installation of the wireless monitoring system rendering it as ideally suited for rapid short-term field studies. Because the wireless communication range in the open field can reach up to 300 m, it was possible to successfully collect data from at least 10 sensors (in this study) simultaneously with a sampling rate of 100 Hz. During data collection, the wireless monitoring system experienced no data loss as a result of a highly robust communication protocol.
2. To autonomously extract the dynamic characteristics of the bridge from structural response time histories, two different approaches were used: the SSI method and the FDD method. The SSI method can provide a good estimation of the number of modes observed in the structure based on singular values of the Hankel matrix projection. On the other hand, the FDD method can only be applied in the frequency domain if the dominant frequencies are determined *a priori*.
3. The results of this test have provided conclusive evidence of the complex dynamic behavior of the bridge. The dynamic response of the cable-stayed bridge is characterized by the presence of many closely spaced, coupled modes. The analytical results of this cable-stayed bridge had been studied before [8]. For most modes, the analytical and the experimental modal frequencies and mode shapes compare quite well. Based on the analysis of ambient vibration data, it is evident that the vertical vibration of the bridge deck is tightly coupled with the cable vibrations within the frequency range of 0 to 3 Hz.

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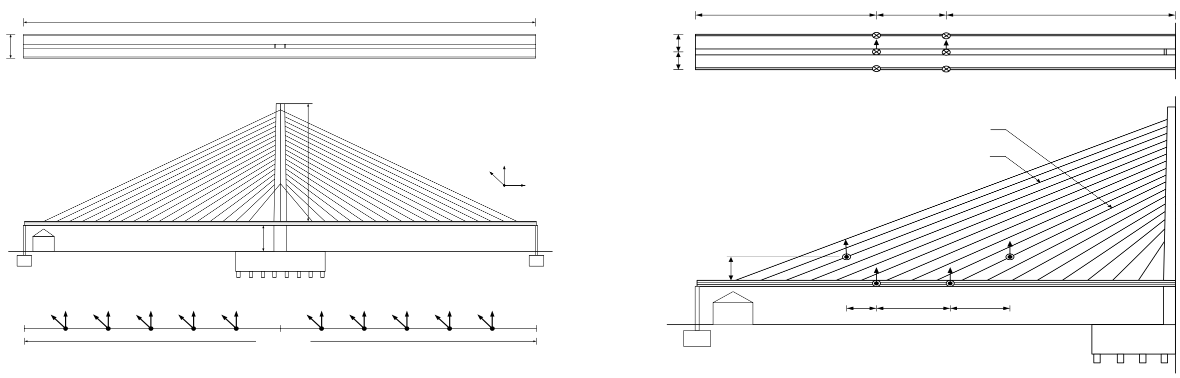
Table 1: Identified natural frequencies from Test-1 and Test-3 using SSI method. The dominant frequencies of cables R-13 and R-27 are also shown.

Test-1* Freq. (Hz)	Test-3* Freq. (Hz)	R-13 Cable** Dominant freq. (Hz)	R-27 Cable** Dominant freq. (Hz)	Note
0.595	0.600	0.600	0.578	1st vertical model freq.
0.985	0.975	0.980	0.980	2nd vertical model freq.
	1.019	/	1.020	R-27 Cable 1st vibration freq.
	1.462	/		Torsion model freq.
1.544	1.539	1.540	1.540	3rd vertical model freq.
	1.809	1.806		R-13 cable 1st vibration freq.
1.853	1.871	1.860	1.860	4th vertical model freq.
2.093	2.029	/	2.027	5th vertical model freq. R-27 cable 2nd model freq.
3.158	/	3.607	3.033	/
4.785	/	5.413	4.053	/
4.850	/	7.227	5.080	/
6.639				

*: identified using stochastic subspace identification method,

** : identified directly from Fourier analysis of measurements,



(1a) (1b)

Figure 1: (a) Front view and top view of the Gi-Lu cable-stayed bridge. Locations of velocity meter-wireless sensor pairs installed along the bridge deck for the ambient vibration survey. (b) Installation location of the wireless sensors during Test 3; velocity meters are installed to record the ambient response of the deck and cables simultaneously.

1. Form the output Hankel matrix Y^s from the measurement sequence y^s according to the user-defined index i.

$$\underset{\sim}{y}^s = \begin{bmatrix} y_0^s & y_1^s & \cdots & y_{j-1}^s & y_j^s & \cdots & y_{2i+j-2}^s \end{bmatrix} \in \mathbf{R}^{l\times r}$$

where $r = 2i + j - 1$

$$Y^s \equiv \left[\begin{array}{cccc} y_0^s & y_1^s & \cdots & y_{j-1}^s \\ y_1^s & y_2^s & \cdots & y_j^s \\ \cdots & \cdots & \cdots & \cdots \\ y_{i-1}^s & y_i^s & \cdots & y_{i+j-2}^s \\ \hline y_i^s & y_{i+1}^s & \cdots & y_{i+j-1}^s \\ y_{i+1}^s & y_{i+2}^s & \cdots & y_{i+j}^s \\ \cdots & \cdots & \cdots & \cdots \\ y_{2i-1}^s & y_{2i}^s & \cdots & y_{2i+j-2}^s \end{array}\right] \equiv \left[\begin{array}{c} Y_p^s \\ \hline Y_f^s \end{array}\right] \in \mathbf{R}^{2li\times j}$$

2. Calculate the orthogonal projection O_i.

$$Y_f^s / Y_p^s \equiv Y_f^s Y_p^{sT} \left(Y_p^s Y_p^{sT} \right)^{\dagger} Y_p^s = O_i^s \in \mathbf{R}^{li\times j}$$

3. Calculate SVD of O_i and determine the order n by neglecting the smaller singular values in S_2.

$$O_i^s = USV^T = \begin{pmatrix} U_1 & U_2 \end{pmatrix} \begin{pmatrix} S_1 & 0 \\ 0 & \cancel{S_2} \end{pmatrix} \begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix} \approx U_1 S_1 V_1^T$$

where $U_1 \in \mathbf{R}^{li\times n}$, $S_1 \in \mathbf{R}^{n\times n}$ and $V_1 \in \mathbf{R}^{j\times n}$

4. Calculate the extended observability matrix Γ_i.

$$\Gamma_i = U_1 S_1^{1/2} \in \mathbf{R}^{li\times n}$$

5. Calculate the system parameter matrices A and C.

$A = \underline{\Gamma}_i^{\dagger} \overline{\Gamma}_i$ and $C =$ the first l rows of Γ_i

where $\underline{\Gamma}_i$ denotes Γ_i without the last l rows

$\overline{\Gamma}_i$ denotes Γ_i without the first l rows

6. Solve the eigenvalue λ_N and Eigenvectors ϕ_{λ_N} of A.

7. Determine the model frequency ω_N and damping coefficient ξ_N from the eigenvalue λ_N.

$$\omega_N = \frac{a_N}{2\pi\Delta t} \ (\mathrm{rad/sec}) \quad \text{and} \quad \xi_N = \frac{|b_N|}{\sqrt{a_N^2 + b_N^2}}$$

where $a_N = \left| \arctan\left(\frac{\mathrm{Im}(\lambda_N)}{\mathrm{Re}(\lambda_N)} \right) \right|$ and $b_N = \ln(\lambda_N)$

8. Calculate the mode

$$\Phi_N = C\phi_N$$

Figure 2: Flow chart of Stochastic Subspace Identification (SSI) technique.

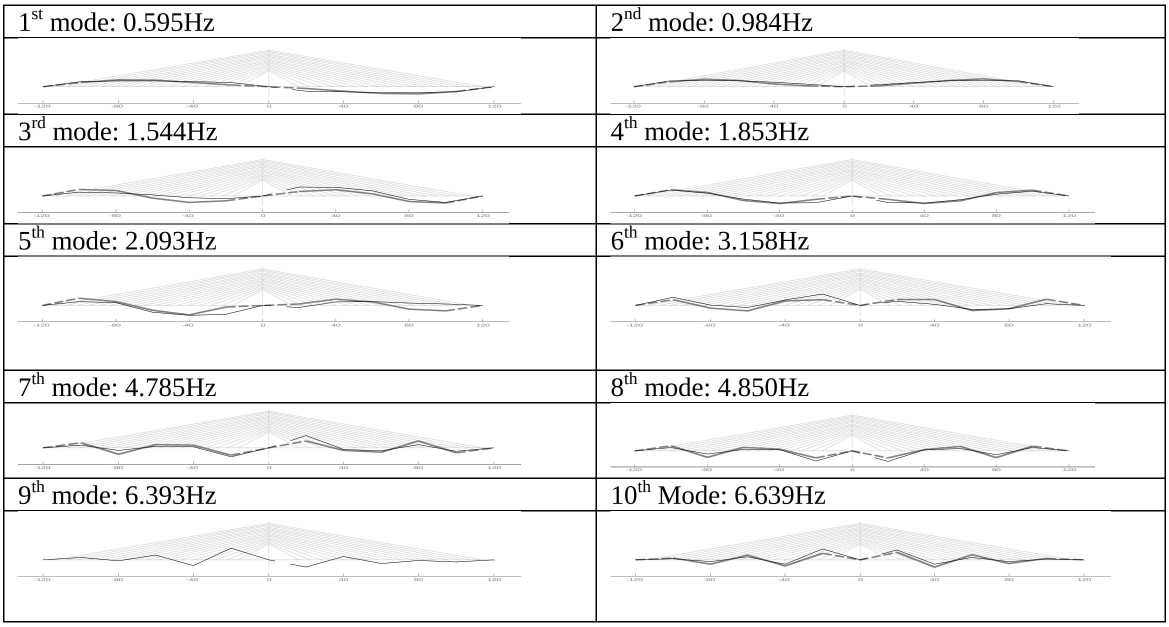


Figure 5a: Comparison of the identified bridge deck vertical mode shapes by using the reference-based stochastic subspace identification and frequency domain decomposition methods.

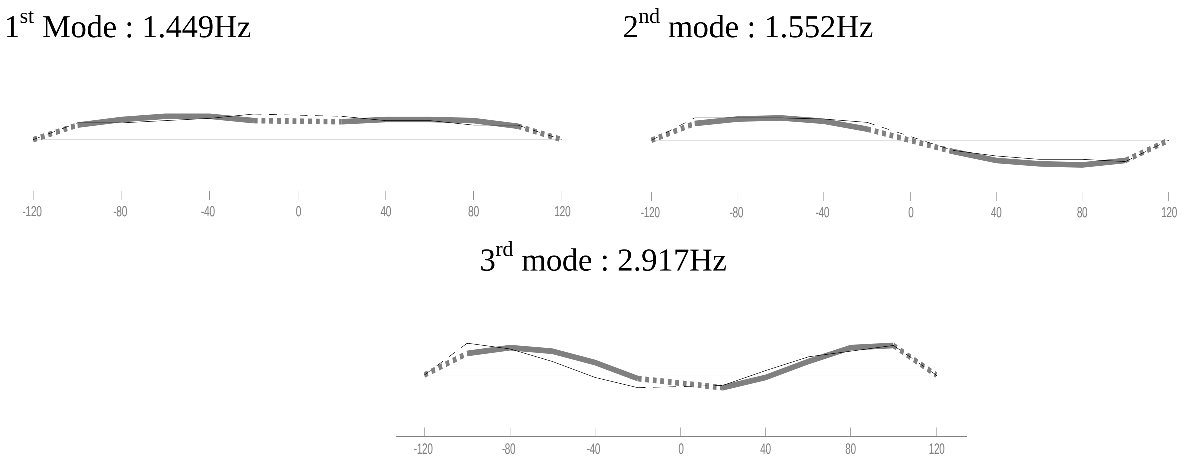


Figure 5b: Comparison of the identified bridge deck mode shapes in the transverse direction by using the reference-based stochastic subspace identification and frequency domain decomposition methods.

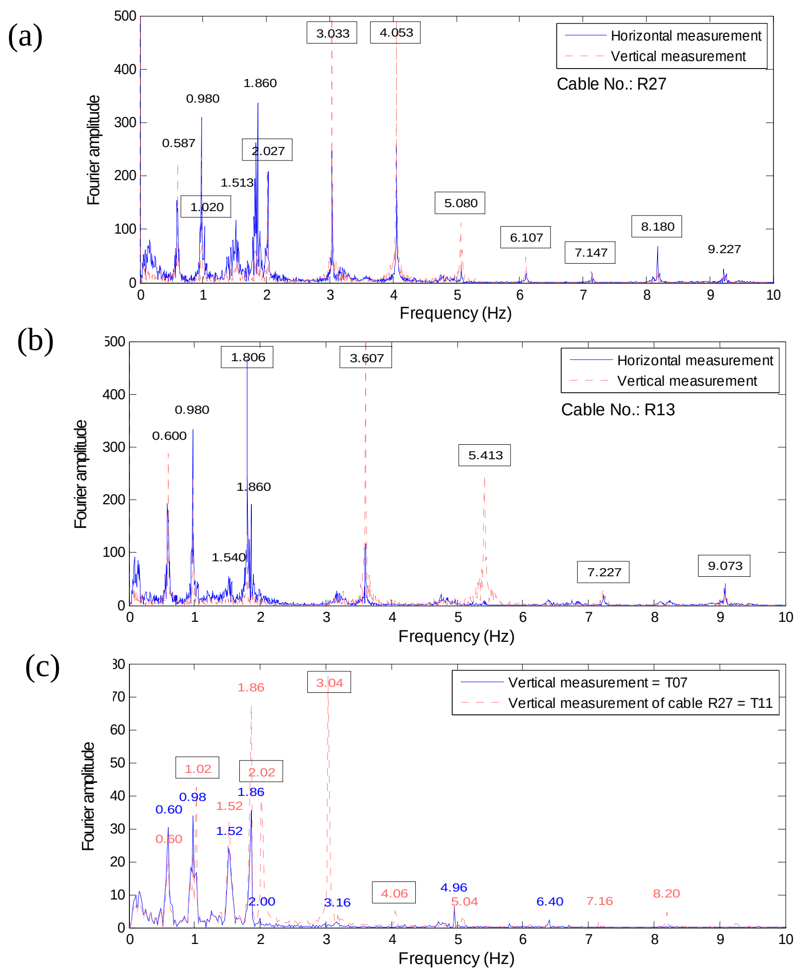


Figure 6: Fourier amplitude spectrum of cable vertical and horizontal vibration data (Fig. 6a for cable R-27 and Fig. 9b for cable R-13). The number in the box is the identified dominant frequency of cable. Comparison on the Fourier amplitude spectrum of cable vibration and deck vertical vibration is shown in Fig.6c.